Statistical methods for estimating historical fire frequency from multiple fire-scar data

William J. Reed and Edward A. Johnson

Abstract: This paper considers the statistical analysis of fire-interval charts based on fire-scar data. Estimation of the fire interval (expected time between scar-registering fires at any location) by maximum likelihood is presented. Because fires spread, causing a lack of independence in scar registration at distinct sites, an overdispersed binomial model is used, leading to a two-variable quasi-likelihood function. From this, point estimates, standard errors, and approximate confidence intervals for fire interval and related quantities can be derived. Methods of testing for the significance of spatial and temporal differences are also discussed. A simple example using artificial data is given to illustrate the computational steps involved, and an analysis of real fire-scar data is presented.

Résumé : Cet article porte sur l'analyse statistique des diagrammes d'intervalle entre les feux basés sur des données de cicatrices laissées par le feu. La méthode du maximum de vraisemblance est utilisée pour estimer l'intervalle entre les feux (période de temps attendue entre les feux qui laissent des cicatrices à n'importe quel endroit). Parce que les feux se propagent, ce qui engendre un manque d'indépendance dans la présence de cicatrices dans différents sites, un modèle binomial exagérément dispersé qui se traduit par une fonction quasi aléatoire à deux variables est utilisé. Les estimations ponctuelles, les erreurs standard et les intervalles de confiance approximatifs pour l'intervalle entre les feux et les quantités qui y sont reliées peuvent être dérivés de cette fonction. Les méthodes qui permettent de tester si les différences spatiales et temporelles sont significatives sont également abordées dans la discussion. Un exemple simple basé sur des données fictives illustre les étapes de calcul et une analyse basée sur de vraies données de cicatrices laissées par le feu est présentée.

[Traduit par la Rédaction]

1. Introduction

Fire-frequency studies have traditionally collected data as time-since-fire maps (Heinselman 1973) or as composite fire-interval charts (Dieterich 1980). Time-since-fire maps have been used in regions in which crown fires predominate, so trees often have only one or rarely a few fire scars. These studies thus consist of a map constructed from fire scars and other evidence of the last fire. After partitioning the map into spatially homogeneous areas, survivorship distributions can be constructed, from which a statistical reconstruction of the fire-frequency history can be obtained, including the identification of change points that separate epochs of assumed constant fire frequency (see Reed (1998, 2000) and Reed et al. (1998) for a discussion of the statistical issues).

In contrast, composite fire-interval charts have been used in regions in which surface fires predominate, so trees usually have multiple scars. These studies consist of a collection of fire-event chronologies based on individual trees with multiple scars or on plots with several trees from which a single fire chronology is constructed. A histogram of fire intervals can be constructed using the data from each chronology. Traditionally, a simple average or median is calculated from the histogram of fire intervals and confidence intervals obtained using a Student *t* procedure. Recently, Grissino-Mayer (1999; see also Johnson 1979) used a Weibull distribution to estimate the fire-frequency parameters.

Several statistical issues are important in the composite fire-interval approach. A proper sampling design must be used in the collection of multiple scar chronologies for any statistical estimate to be valid. In other words, every possible chronology must have an equal chance of being chosen in a sample of chronologies. One cannot just choose trees or plots with the most scars or those that are easily accessible (Johnson and Gutsell 1994). Also not all trees are scarred in a particular fire. Baker and Ehle (2001) have discussed this and other concerns with field methods, data collection and processing.

The traditional method of simply calculating a Student *t* confidence interval using the observed intervals between scars on all trees in the sample, while easy to compute, is not really valid. The assumptions behind the Student *t* procedure are that the data are independent observations from a normal distribution. Both of these assumptions are likely violated for fire-interval data. First their distribution will typically not be normal. This can be seen in Fig. 1, which presents a frequency plot of all intervals between scars on individual trees for the Dugout region of the Blue Mountains in eastern Oregon (see Sec. 4.2). The data are clearly not normally distributed. Indeed their distribution looks closer to an exponential distribution, which is what would be expected with a constant hazard of burning. A second and probably more serious violation of assumptions concerns that of independence, for example, two successive fires may

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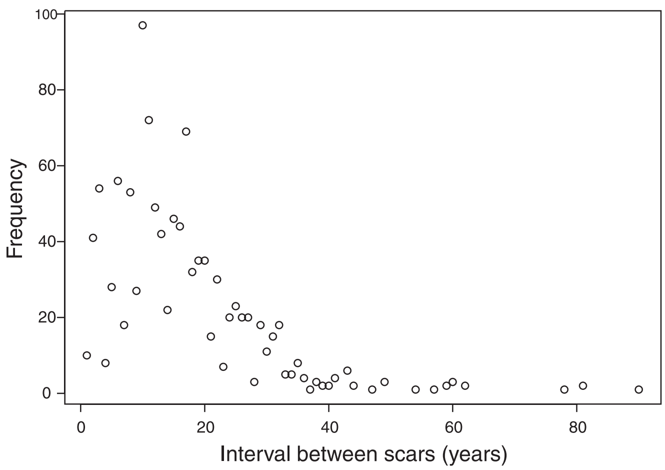
Fig. 1. A frequency plot of intervals between scars on all sample objects in the Dugout region of the Blue Mountains in eastern Oregon. Note how the distribution is far from normal (as required for the validity of the Student t procedure).



both be recorded on each of two (or more) separate sample objects, leading to two (or more) identical fire intervals. While the lack of normality may not greatly affect point estimates, lack of independence certainly can, and both violations of assumptions will render confidence intervals invalid.

The objective of this paper is to remedy the shortcomings in the traditional procedure by developing a statistical methodology, based on the maximum likelihood paradigm for analyzing composite fire-interval charts, in particular for estimating (with point estimates and confidence intervals) the expected time between fires at any location or its inverse, the fire frequency. The main novelty of the procedure involves incorporating into the analysis the fact that the same fire may register scars on several sample objects. This is achieved by developing a model in which the occurrence of fires and the spread of fires are handled separately. The null model of survival analysis (a constant hazard rate) is used for the former, while the contagious effect of fire spread is handled by using an overdispersed binomial distribution. For such a model, the probability of any object recording a scar is the same, but these events are assumed to not be independent, with contagion present. Because the number of sample objects vulnerable to scarring changes over time, in order to use the overdispersed binomial distribution, the period over which observations are made must be divided into nonoverlapping epochs, within which the number of vulnerable sample objects remains constant. These ideas are developed in greater detail in the following sections.

The paper starts by establishing a terminology and notation (Sec. 2). In Sec. 3, a model is developed and estimation by maximum likelihood discussed. Methods for testing for differences (both spatial and temporal) in fire frequency are also discussed. In Sec. 4, a simple example using artificial data is given to illustrate the calculations involved, and this is followed by a more complete example using real data kindly made available by E. Heyendahl (Heyendahl et al. 2001).

For the reader's convenience, a list of symbols and their meanings is given at the end of the References.

2. Definitions and notation

Typically, fire-scar data will come from a number of sites at which dendrochronological observations are made on sampled trees as well as possibly on other objects, such as logs, stumps, snags, etc. Because sampled trees likely will have originated at different times (and logs, stumps, etc. ceased growing at different times), sampled objects in general will have been vulnerable to scarring over different periods. For the purpose of analysis, we shall consider the past as divided into distinct epochs, during each of which a constant number of sampled objects are assumed to have been vulnerable to scarring. Thus the first (oldest) epoch will comprise the time from the date of establishment of the oldest sampled object to the date of its demise or to the establishment of the next oldest sampled object, whichever is earlier. During this period only one object will have been vulnerable. The next epoch, during which one or two sampled objects will have been vulnerable, will comprise the time between the establishment of the second oldest object and either the establishment of the third oldest object or the death of one of the previously established objects. In general, we shall suppose that there are M epochs, which, if we set as the time origin the date of establishment of the oldest sampled object, comprise the time intervals $0 - T_1, T_1 - T_2, \ldots, T_{M-1} - T_M$.

Let the number of objects vulnerable to scarring during epoch j be denoted by N_j, ($j = 1, \ldots, M$). A special case is when all sampled objects are live trees, originating at distinct dates. In this case $N_1 = 1, N_2 = 2, \ldots, N_M = M$. More generally, the sequence $\{N_j\}$ will increase (or decrease) between epochs separated by the establishment (or death) of an object. Let the number of distinct dates at which fires were recorded during epoch j be denoted by n_j, and let the numbers of scars on sampled objects recorded at each of these dates be denoted by $x_{j,1}, x_{j,2}, \ldots, x_{j,n_j}$, respectively. Thus, during epoch j there will be $x_{j.} = \sum_{r=1}^{n_j} x_{j,r}$ scars recorded, providing evidence of at least n_j fires during that epoch.

We note that if more than one scar is registered at any time, it will be assumed that the scars were caused by the same fire. Without more complete geographical information, there is no way to distinguish separate fires that occur in the same year.

3. Model, assumptions, and maximum likelihood estimation

To analyze data of the type described previously, it is necessary to make some assumptions about the way in which the data were generated. Thus we assume that the study area is homogenous with respect to fire hazard, and that this has been unchanging over time. (Later we relax these assumptions and allow for different hazards in different subregions and also allow a temporally varying hazard that is constant over intervals separated by change points.) We model this by assuming that there is an unchanging area-wide hazard of scarring, λ; that is, we assume that the probability of a fire, which registers a scar somewhere in the study area during an infinitesimal time interval $(t, t+h)$, is $\lambda h + o(h)$ for all t, $0 \leq t \leq T_M$. (Note that the term "hazard of burning" was used to denote the per-annum probability of fire at a location computed instantaneously, that is, over an infinitesimal interval; see Johnson and Gutsell (1994) and Reed et al. (1998). Here, the term "area-wide hazard of scarring" is used to denote the per-annum probability of a fire leaving a scar somewhere in the study area.)

If such a fire occurs, it may or may not leave a scar on any particular sample object. Assume that the probability that a scar-registering fire in the study area leaves a scar on a given sample object is the same for all sample objects and denote this probability p, and let $q = 1 - p$. Thus the hazard of scarring for a particular sample object is $\theta = \lambda p$ (the same for all sample objects). We shall refer to θ as the local hazard of scarring. Its reciprocal is the expected time between scar-causing fires (fire interval) at any location. Our primary objective will be to estimate θ and the fire interval $\mathrm{FI} = 1/\theta$.

We now need to consider the distribution of the number of scars registered for a particular fire. If a given fire did or did not leave a scar on a vulnerable object, independently of what happened on other vulnerable objects, then with N vulnerable objects, the number of scars registered would follow a binomial $B(N, p)$ distribution truncated on $x = 1, 2, \ldots, N$ (i.e., excluding 0). However, the assumption of independence is unrealistic; given the fact that fires spread spatially, there will be contagion present in the distribution. The presence of a contagious effect can be detected statistically by testing whether the numbers of scars registered for each fire in an epoch conform to a binomial distribution against the alternative of overdispersion, using a binomial dispersion test (e.g., Kendall and Stuart 1967). The test statistic is

$$[1] \qquad D = \frac{(n-1)s^2}{\bar{x}(1 - \bar{x}/N)}$$

where $\bar{x}$ and s^2 are the sample mean and variance of the numbers of scars registered for each of the n fires in the epoch, respectively, and N is the number of vulnerable objects . Under the null hypothesis of no contagion $D \sim \chi^2_{n-1}$ asymptotically. To demonstrate the presence of contagion, we carried out this test for all epochs with two or more fires for data on the Dugout region of the Blue Mountains in eastern Oregon (see Sec. 4.2 and Table 1). It can be seen that, for all (seven) epochs with five or more fires, the P value was extremely small (much less than 0.0001). The only epochs for which it is not highly significant are those with very few fires. The test is of low power in such cases, so this is not surprising. However, in spite of this, the test was highly significant for three of the four epochs with only two fires. One can easily see the overdispersion in these cases. Consider, for example, Epoch 12 when 69 sample objects were vulnerable and two fires occurred, registering 1 and 44 scars, respectively. This is extremely unlikely if scars were independently registered on distinct objects. Rather, there is overdispersion resulting from the second fire spreading extensively and the first not doing so. Thus we have strong evidence of contagion or overdispersion and need a distribution that reflects this fact.

An alternative formulation that allows for contagion effects is to assume that the number of scars registered follows what is known as an overdispersed form of the (zero-truncated) binomial distribution (see, e.g., Pawitan 2001, p. 76). Such a distribution involves an dispersion parameter ϕ, along with the binomial parameters N and p. Its mean is the same as that of the zero-truncated binomial, but its variance is inflated by a factor ϕ, which reflects the degree of contagion in the formation of scars on sample objects. The case $\phi = 1$ corresponds to independence (no contagion), with ϕ increasing with the degree of contagion.

Table 1. Data and binomial dispersion test for scars in Dugout region.

Epoch j	No. of objects N_j	No. of fires n_j	No. of scars $x_{j,r},\ r = 1, \ldots, n_j$	P value
1	53	3	51, 1, 1	<0.0001
4	59	2	2, 1	0.56
10	67	3	1, 1, 2	0.48
11	68	4	1, 1, 1, 5	0.10
12	69	2	1, 44	<0.0001
13	70	7	5, 2, 1, 1, 2, 1, 57	<0.0001
14	71	5	8, 1, 29, 1, 64	<0.0001
15	72	10	1, 3, 23, 2, 66, 1, 9, 1, 1, 7	<0.0001
16	71	8	16, 8, 12, 7, 36, 2, 1, 60	<0.0001
17	70	6	2, 3, 22, 31, 12, 51	<0.0001
18	68	3	1, 3, 32	<0.0001
19	66	10	27, 2, 47, 1, 5, 3, 21, 23, 1, 35	<0.0001
20	65	5	11, 6, 54, 1, 47	<0.0001
24	56	3	5, 4, 7	0.62
25	53	2	2, 21	<0.0001
29	38	2	3, 16	0.0006
34	12	3	2, 1, 5	0.14

Note: All epochs with two or more fires are included. The null hypothesis is that the number of scars is binomially distributed.

An advantage of using such a distribution is that it is a member of the exponential dispersion family (see, e.g., Pawitan 2001, p. 97), whose properties are well understood and for which estimation procedures have been developed. To do this one constructs a quasi-likelihood function that, at least for inference for parameters other than the dispersion parameter ϕ, can be treated like an ordinary log-likelihood. To this end, we calculate first the probability of observing the given data (which comprises times and numbers of scars registered for each fire). Since events in distinct epochs are independent, the probability of observing the full data can be expressed as

$$[2] \qquad \Pr(\text{observed data}) = \prod_{j=1}^{M} \Pr(\text{observed data in epoch } j)$$

To evaluate this further, consider a generic epoch of duration τ with N sample vulnerable objects. (Note that while discussing a generic epoch we suppress the epoch-identifying subscript j). Suppose that scars were left at n distinct dates, $t_1, t_2, \ldots, t_n$ time units after the start of the epoch, with x_i, $(i = 1, 2, \ldots, n)$ scars left at time t_i. We can write

$$\Pr(\text{observed data}) = \Pr(x_1, x_2, \ldots, x_n \text{scars registered}|\text{fires at } t_1, t_2, \ldots, t_n)$$

$$\Pr(\text{fires occurred at } t_1, t_2, \ldots, t_n) = P_{x|t} P_t$$

Consider first the probability P_t. Under the assumed model, the probability (density) of observing fire-registering scars at times $t_1, t_2, \ldots, t_n$, in the study area, with no fires registered at other times, can be obtained as the product of exponential densities for times between fires multiplied by the probability of no fire between t_n and τ. Precisely

$$[3] \qquad P_t = \left[\lambda e^{-\lambda t_1}\right]\left[\lambda e^{-\lambda(t_2-t_1)}\right] \times \left[\lambda e^{-\lambda(t_3-t_2)}\right]\ldots\left[\lambda e^{-\lambda(t_n-t_{n-1})}\right]\left[e^{-\lambda(\tau-t_n)}\right] = \lambda^n e^{-\lambda\tau}$$

At time t_1, the probabilty of x_1 scars being registered is given by the probability mass function (pmf) $f(x_1; N, p, \phi)$ of the overdispersed zero-truncated binomial distribution. Thus the probability of $x_1, x_2, \ldots, x_n$ scars being observed, conditional on fires occurring at times $t_1, t_2, \ldots, t_n$, is

$$[4] \qquad P_{x|t} = \prod_{r=1}^{n} f(x_r; N, p, \phi)$$

so that for the epoch

$$[5] \qquad \Pr(\text{observed data}) = \lambda^n e^{-\lambda\tau} \prod_{r=1}^{n} f(x_r; N, p, \phi)$$

and for the full data set

$$[6] \qquad \Pr(\text{observed data}) = \lambda^{n\cdot} e^{-\lambda T} \prod_{j=1}^{M}\prod_{r=1}^{n_j} f(x_{j,r}; N_j, p, \phi)$$

where $T = T_M$ is the full time for which observations are available and $n. = \sum_{j=1}^{M} n_j$ is the total number of fires over that period. To construct a quasi-likelihood it is not necessary to have an explicit expression for $f(x; N, p, \phi)$. Rather all we need to know is that its logarithm is of the form (see, e.g., Pawitan 2001)

$$[7] \qquad \log(f(x; N, p, \phi)) = \frac{x\log(p/q) + \log q^N - \log(1-q^N)}{\phi} + c(\phi, \text{data})$$

where $q = 1-p$ and $c(\phi, \text{data})$ does not depend on the parameters λ and p. Note that the numerator of the first term is the logarithm of the zero-truncated binomial pmf $\binom{N}{x} p^x q^{n-x}/(1-q^N)$ apart from the constant term not involving p, which is absorbed into the $c(\phi, \text{data})$ term in eq. 7. In particular, with $\phi = 1$, eq. 8 is simply the log-likelihood for one observation from a zero-truncated binomial distribution. The more general form (with ϕ unspecified) allows for overdispersion in the zero-truncated binomial distribution.

Taking the logarithm of eq. 7 (and ignoring terms involving only ϕ and the data) one gets the quasi-likelihood

$$[8] \qquad Q = n.\log\lambda - \lambda T + \left(\frac{1}{\phi}\right)\left[x..\log\left(\frac{1-q}{q}\right) + \sum_{j=1}^{M} n_j\left(\log q^{N_j} - \log(1-q^{N_j})\right)\right]$$

where $x.. = \sum_{j=1}^{M}\sum_{r=1}^{n_j} x_{j,r}$ is the total number of scars observed for the study and $n = \sum_{j=1}^{M} n_j$ is the total number of fires observed. Note that Q is not a full log-likelihood because it does not include the contribution of the parameter ϕ via the term $c(\phi, \text{data})$; however, it correctly includes the contributions to the log-likelihood estimate of the other parameters λ and p (via q). To obtain maximum likelihood estimates (MLEs) of λ and q one can set the derivatives of Q with respect to λ and q equal to zero. This leads to the following estimating equations for the MLEs of λ and q:

$$[9] \qquad \lambda = n./T \qquad x.. = (1-q)\sum_{j=1}^{M}\frac{n_j N_j}{1-q^{N_j}}$$

The second (polynomial) equation in q needs to be solved numerically. The first yields the MLE of the area-wide hazard of scarring λ as simply the number of fires producing scars observed per unit time. The MLEs $\hat{q}$ and $\hat{\lambda}$ are independent.

To estimate the dispersion parameter ϕ, a moment estimator can be used (see, e.g., Patiwan 2001, p. 165). This yields

$$[10] \qquad \hat{\phi} = \frac{1}{n.-1}\sum_{j=1}^{M}\frac{1}{V(\hat{q}, N_j)}\sum_{r=1}^{n_j}\left[x_{j,r} - \frac{N_j(1-\hat{q})}{1-\hat{q}^{N_j}}\right]^2$$

where

$$[11] \qquad V(q, N) = N\frac{q(1-q)}{1-q^N}\left[1 - \frac{N(1-q)q^{N-1}}{1-q^N}\right]$$

is the variance of the zero-truncated binomial distribution. (Note that when $N_j = 1$ and $n_j = 1$, both the numerator and denominator of the summand (at j) in eq. 11 are zero. In this case, since there is clearly no overdispersion, the summand is one. Also when $n_j = 0$ the summand is zero.) To compute the sums of squares in eq. 11 it may be more convenient to use the alternative form

$$[12] \qquad \sum_{r=1}^{n_j} x_{j,r}^2 - 2\frac{N_j(1-\hat{q})}{1-\hat{q}^{N_j}}\sum_{r=1}^{n_j} x_{j,r} + \frac{n_j N_j^2(1-\hat{q})^2}{(1-\hat{q}^{N_j})^2}$$

The MLE of the local hazard of scarring is $\hat{\theta} = \hat{\lambda}\hat{p} = \hat{\lambda}(1-\hat{q})$ and its reciprocal $1/\hat{\theta}$ is the MLE of the fire interval FI (expected time between fires at any given location).

The standard error of the MLE $\hat{\lambda}$ can be computed (as the square root of the inverse of the observed information) as

$$[13] \qquad s_{\hat{\lambda}} = \sqrt{n.}/T.$$

In a similar fashion the standard error of $\hat{q}$ can be computed:

$$s_{\hat{q}} = \sqrt{\hat{\phi}}\left[\frac{x..}{(1-\hat{q})^2} + \frac{\sum_{j=1}^{M} n_j N_j - x..}{\hat{q}^2} + \sum_{j=1}^{M}\frac{n_j N_j \hat{q}^{N_j-2}\left(N_j - 1 + \hat{q}^{N_j}\right)}{(1-\hat{q}^{N_j})^2}\right]^{-1/2}$$

and then the standard error of $\hat{\theta}$ can be calculated using

[14] $$s_{\hat{\theta}} = \left[s_{\hat{\lambda}}^2 s_{\hat{q}}^2 + (1-\hat{q})^2 s_{\hat{\lambda}}^2 + \hat{\lambda}^2 s_{\hat{q}}^2 \right]^{1/2}$$

The standard error of the fire interval can be calculated (from the observed information after reparameterization, or by the delta-method) as

[15] $$s_{\hat{\mathrm{FI}}} = \frac{1}{\hat{\theta}} \left[\frac{s_{\hat{\lambda}}^2}{\hat{\lambda}^2} + \frac{s_{\hat{q}}^2}{(1-\hat{q})^2} \right]^{1/2}$$

and a $100(1-\alpha)\%$ confidence interval for the fire interval found as $\hat{\mathrm{FI}} \pm z_{\alpha/2} s_{\hat{\mathrm{FI}}}$, where $z_{\alpha/2}$ is the $100(\alpha/2)$ percentage point of the standard normal distribution.

For computing a P value for testing the equality of the fire interval in two distinct regions, one can compare the observed value of the test statistic

[16] $$\frac{\hat{\mathrm{FI}}_1 - \hat{\mathrm{FI}}_2}{\sqrt{s_{\hat{\mathrm{FI}}_1}^2 + s_{\hat{\mathrm{FI}}_2}^2}}$$

with a standard normal distribution.

3.1. Testing for temporal changes

It is straightforward to test whether the fire interval changed at any prespecified time (e.g., time of settlement by Europeans): one can simply divide the data into two parts, before and after the hypothesized change point, and compute a P value using the test statistic given in eq. 16. However, if one wishes to use scar data to identify change points, one faces the same selection-bias problems that one does when using time-since-fire data (Reed et al. 1998). To overcome that problem, two methods were proposed by Reed (1998, 2000), the first based on an iterative stepwise procedure and the second on the use of the Bayes' information criterion (BIC). While application of the first method to scar data is not immediately obvious, that of the second should be straightforward.

4. Examples

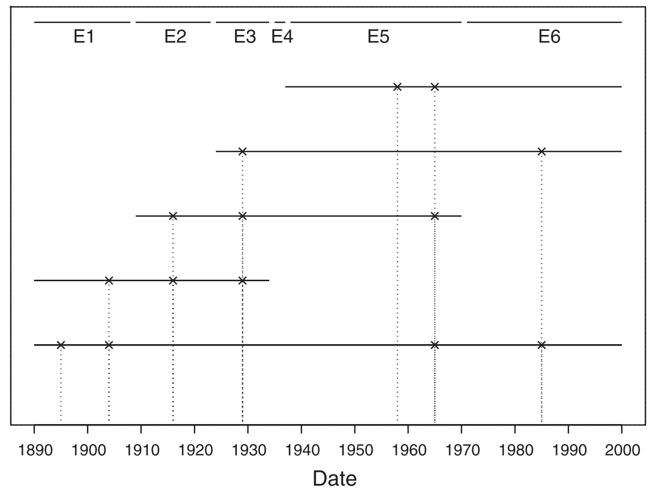
In this section two examples are given. The first uses a very simple artificial data set and is presented to illustrate the calculations required. The second uses real data for the Blue Mountains of eastern Oregon.

4.1. Artificial data

Figure 2 shows (fake) data for fire scars occurring over a 110-year period. Five sample objects (represented by horizontal lines) exhibit scars (represented by ×'s). One commenced in 1890 and was still extant in 2000; another commenced in 1890 but was not present beyond 1934, etc.

To identify the epochs for these data, we start at 1890 and observe that there were two objects vulnerable until the origin of a new sample tree in 1910. Thus the first epoch is 1890–1909 with $N_1 = 2$ sample objects and $n_1 = 2$ fires (in 1895 and 1904). The earlier fire left $x_{1,1} = 1$ scar, and the later one left $x_{1,2} = 2$ scars. The second epoch is from 1910 to 1925, when a new sample tree originated. In this epoch there were $N_2 = 3$ sample objects and $n_2 = 1$ fires (in 1916), which left $x_{2,1} = 2$ scars. Continuing in this way one finds six epochs in the time period 1890–2000 ($T = 110$), shown at the top of Fig. 1 and labelled E1–E6. Details are given in Table 2.

Fig. 2. A composite fire-interval chart (artificial data) for the example of Sec. 4.1. There are five sample objects: two originated in 1890, one in 1909, one in 1924, and the last in 1937. Of these, all but two were still in existence in 2000. Fire scars are marked by crosses, and the distinct epochs shown at the top of the figure are marked as E1, E2, etc.



The total number of distinct fires is $n_{\cdot} = 7$. All together they registered $x_{\cdot\cdot} = 15$ scars. The MLE of the area-wide hazard of scarring for all sample objects is $\hat{\lambda} = 7/110 = 0.064$. The MLE of $q = 1 - p$ is found by solving eq. 10

$$\frac{15}{1-q} = \frac{4}{1-q^2} + \frac{6}{1-q^3} + \frac{12}{1-q^4}$$

which yields the solution $\hat{q} = 0.3475$ with the corresponding MLEs $\hat{p} = 0.6525$, $\hat{\theta} = 0.0415$, and $\hat{\mathrm{FI}} = 24.08$ years. From eq. 11, the dispersion parameter is estimated as $\hat{\phi} = 1.224$. The SE of the estimate of the fire interval is 9.91 years, yielding a 95% confidence interval of 4.7–43.5 years.

For comparison purposes we note that the mean (and SD) of the nine observed interscar intervals is 25.22 (and 20.74) years. A 95% confidence interval based on an assumed t_8 distribution is (−22.6, 73.0) or 0 to 73.0 years. It can be seen then that, in this example, the "traditional" method of estimation yields an estimate close to the new method, but a very different confidence interval.

4.2. Blue Mountain data

For a second example we use real data collected in the Blue Mountains of eastern Oregon, USA, by E.K. Heyerdahl (Heyerdahl 1997; Heyerdahl et al. 2001). We use four sites: Tucannon and Imnaha (both of which have north- and south-facing hillslopes), Baker (northeast-facing hillslopes) and Dugout (west-facing hillslope).

The south-facing slopes of Tucannon and Imnaha have dry forests dominated by open forests of Douglas-fir (*Pseudotsuga menziesii* (Mirb.) Franco) and pine grass (*Calamagrostis*

Table 2. Fake data (shown graphically in Fig. 1) used for illustrating calculations in Sec. 4.1.

	Epoch j					
	1	2	3	4	5	6
	1890–1909	1910–1925	1926–1934	1935–1937	1938–1970	1971–2000
N_j	2	3	4	3	4	3
n_j	2	1	1	0	2	1
$t_{j,r}$	5, 14	26	39	—	68, 75	95
$x_{j,r}$	1, 2	2	3	—	3, 1	3

Table 3. Estimates of the fire interval for sites in the Blue Mountains.

Site (aspect)	MLE of FI (years)	SE	Estimated dispersion, $\hat{\phi}$	95% CI for FI	Mean (years)	95% Student t CI for FI
Tucannon (N)	183.5.0	102.3	6.92	0–384.0	102.6	47.2–158.0
Tucannon (S)	42.2	8.8	8.05	24.9–59.4	34.0	0–88.6
Imnaha (N)	118.2	79.8	21.16	0–274.6	50.3	12.9–87.6
Imnaha (S)	34.2	13.23	57.32	8.2–60.1	26.0	0–55.4
Baker (NE)	23.0	3.78	9.84	15.6–30.4	16.1	0–47.7
Dugout (W)	21.7	3.65	28.06	14.5–28.8	15.6	0–35.6

Note: The penultimate column is the mean of all observed interscar intervals, which has been suggested as an estimator of FI. The the last column is a 95% Student t confidence interval (CI) based on observed interscar intervals. (Note that for all confidence intervals if the lower limit is negative it is reported as zero.)

Table 4. Estimates of the fire interval for three epochs (late: 1890–1994; middle: 1730–1889; early: pre-1730) in dry sites in the Blue Mountains.

Site (aspect)	Epoch	MLE of FI (years)	SE[a]	Estimated dispersion, $\hat{\phi}$	95% CI[b] for FI
Baker (NE)	Late	87.4	71.60	20.59	0–227.7
	Middle	22.3	5.72	10.34	11.1–33.5
	Early	15.7	3.46	8.00	8.9–22.5
Dugout (W)	Late	35.9	17.29	41.66	2.0–69.8
	Middle	13.8	3.08	29.85	7.8–19.9
	Early	26.9	7.73	13.38	11.7–42.0
Tucannon (S)	Late	68.4	43.96	21.53	0–154.5
	Middle	22.4	5.21	5.33	12.2–32.6
	Early	68.3	28.50	2.86	12.4–124.1
Imnaha (S)	Late	30.9	36.23	197.42	0–101.9
	Middle	48.4	23.10	21.93	3.1–93.7
	Early	37.8	9.73	4.13	18.7–56.9

[a]SE, standard error.
[b]CI, confidence interval.

rubescens Buckl.) with some grand fir (*Abies grandis* (Dougl.) Forbes. The north-facing slopes have mesic forest dominated by grand fir and huckleberry (*Vaccinium membranaceum* Dougl.), and at higher elevations in Tucannon there is some subalpine fir (*Abies lasiocarpa* (Hook.) Nutt.) and huckleberry (*Vaccinium* spp.). The Dugout and Baker sites are almost completely dry forest of Douglas-fir and pine grass with some grand fir. Baker has a mesic forest, with subalpine fir at higher elevations.)

Each site was divided into cells each approximately 25 ha. A 1-ha plot was placed in the center of each cell. A fire-event chronology was contracted from fire scars and tree ages for each 1-ha plot. The south-facing and north-facing parts of the Tucannon and Imnaha sites are treated separately for analysis, making six study areas in all. Table 3 gives estimates of the fire interval in the six areas. Also given in Table 3 (last two columns) is a point estimate using the mean of all observed interscar intervals and a 95% confidence interval using a Student t procedure. Notice how this method produces estimates lower than the MLEs obtained using the method established in this paper. Indeed, in the two cases with low fire incidence (Tucannon (N) and Inmaha (N)) the MLEs of the fire interval are larger than the mean estimates by a factor of about two and lie outside (above) the Student t confidence intervals.

It appears the sites cluster into three sets of two (Baker and Dugout; south-facing slopes of Imnaha (S) and Tucannon (S); and north-facing slopes of Imnaha (N) and Tucannon (N)). The only significant differences using the statistic displayed in eq. 16 are between Tucannon (S) and (*i*) Dugout ($P = 0.03$) and (*ii*) Baker ($P = 0.04$). (Note that because multiple comparisons are being considered, these tests should be seen only as guides and not be interpreted too literally.) Although the estimates of the fire cycle for the north-facing slopes of Tucannon and Imnaha are considerably larger than those of the other sites, they do not show up as significantly different, because of the large standard errors associated with the estimates, which are based on very few fires.

Many other studies have shown temporal changes in the fire cycle. These can be tested in the fashion described in Sec. 3.1, by dividing the data into the epochs defined by the hypothesized change points. Earlier studies (Heinselman 1973; Johnson et al. 1990; Masters 1990; Bergeron and Archambault 1993; Yarie 1998; Weir et al. 2000) suggest that the 1890s and 1730s marked changes in the fire regime. Thus the following three epochs were considered: (*i*) pre-1730, (*ii*) 1730–1889, and (*iii*) 1890–1994. Table 4 gives estimates of the fire cycle for these three epochs in the four dry regions.

For Baker and Dugout, the estimates of the fire cycle for the early and late periods are longer than those for the middle period. However, in no case is the difference strongly significant (the strongest evidence of a difference is between early and middle periods for Tucannon and Dugout, both with (one-sided) $P = 0.06$). The common pattern exhibited in the three regions suggests that the lack of evidence of differences could be due to the poor power of the test, because of the relatively small numbers of fires recorded. This is especially true of the late periods, for which the standard errors of estimates of the fire cycle are very large. The Inmaha sites exhibit a temporal pattern different from the other three, with the estimates of the fire cycle in the middle period being longer than those in the early and late periods.

5. Conclusions

This paper presents, for the first time, sound statistical methods for analyzing fire-history studies from ecosystems with multiple-scarred trees. Using these methods along with a statistically valid sampling design will help in evaluating the historic range of variations of fire in a surface-fire system such as open-canopied ponderosa pine and Douglas-fir forests.

One of the most important points revealed in the application of the method is that, in many multiple-scarred tree fire-history studies, the sample of chronologies is too small to draw unambiguous conclusions, a point made earlier by Baker and Ehle (2001). This limitation can be seen in the Heyerdahl et al. (2001) study, where, even though a large number of fires burned the whole study area, confidence intervals are still quite wide in some instances. If the sample area is further divided to study spatial and (or) temporal changes, this problem is exacerbated.

It has been claimed that there is a significant problem in composite fire-interval studies in that, as the sample size increases, the estimate of the mean fire interval decreases towards one (a fire once a year), simply because evidence of more fires is found as more trees and objects are sampled (Arno and Petersen 1983; Baker and Ehle 2001). This difficulty emanates from the lack of distinction between the area-wide hazard λ and the local hazard $\theta = \lambda p$ and their reciprocals (area-wide and local fire intervals). The estimate of the area-wide fire interval would indeed tend downwards as the number of sampled objects increased, but it is not true that estimates of the local fire interval would necessarily decrease (because the effect on the estimate of the parameter p could be in either direction). However, in concordance with the usual results of increasing sample size, the standard error of the estimate of the local fire interval would decrease.

References

Arno, S.F., and Petersen, T.D. 1983. Variation in estimates of fire intervals: a closer look at fire history on the Bitterroot National Forest. USDA For. Serv. Res. Pap. INT-301.

Baker, W.L., and Ehle, D. 2001. Uncertainty in surface-fire history: the case of ponderosa pine forests in the Western United States. Can. J. For. Res. **31**: 1205–1226.

Bergeron, Y., and Archambault, S. 1993. Decreasing fire frequency of forest fires in the southern boreal zone of Quebec and its relationship to global warming since the end of the "Little Ice Age". Holocene, **3**: 255–259.

Dieterich, J.H. 1980. The composite fire interval: a tool for more accurate interpretation of fire history. *In* Proceedings of the fire history workshop. USDA For. Serv. Gen. Tech. Pap. RM-81. pp. 8–13.

Grissino-Mayer, H.D. 1999. Modeling fire interval data from the American Southwest with the Weibull distribution. Int. J. Wildland Fire, **9**: 37–50.

Heinselman, M.L. 1973. Fire in the virgin forests of the Boundary Waters Canoe Area. Quat. Res. **3**: 329–382.

Heyerdahl, E.K. 1997. Spatial and temporal variation in historical fire regimes of the Blue Mountains, Oregon and Washington: the influence of climate. Ph.D. thesis, University of Washington, Seattle.

Heyerdahl, E.K., Brubaker, L.B., and Agee, J.K. 2001. Spatial controls of historical fire regimes: a multiscale example from the Interior West, U.S.A. Ecology, **82**: 660–678.

Johnson, E.A. 1979. Fire recurrence in the subarctic and its implications for vegetation composition. Can. J. Bot. **57**: 1374–1379.

Johnson, E.A., and Gutsell, S.L. 1994. Fire frequency models, methods and interpretations. Adv. Ecol. Res. **25**: 239–287.

Johnson, E.A., Fryer, G.I., and Heathcott, M.J. 1990. The influence of man and climate on frequency of fire in the interior wet belt forest, British Columbia. J. Ecol. **78**: 403–412.

Kendall, M.G., and Stuart, A. 1967. The advanced theory of statistics, Vol. 2. 2nd ed. C. Griffin and Co., London.

Masters, A.M. 1990. Changes in forest fire frequency in Kootenay National Park, Canadian Rockies. Can. J. Bot. **68**: 1763–1767.

Pawitan, Y. 2001. In all likelihood: statistical modelling and inference using likelihood. Oxford University Press, Oxford.

Reed, W.J. 1998. Determining changes in historical forest fire frequency from a time-since-fire map. J. Agric. Biol. Environ. Stats. **3**: 430–450.

Reed, W.J. 2000. Reconstructing forest fire history-identifying hazard rate change points using the Bayes' information criterion. Can. J. Stat. **28**: 352–365.

Reed, W.J., Larsen, C.P.S., Johnson E.A., and MacDonald, G.M. 1998. Estimation of temporal variations in historical fire frequency from time-since fire map data. For. Sci. **44**: 465–475.

Weir, J.M.H., Johnson, E.A., and Miyanishi, K. 2000. Fire frequency and the spatial age mosaic of the mixedwood boreal forest in Western Canada. Ecol. Appl. **10**: 1162–1177.

Yarie, J. 1998. Forest fire cycles and life tables: a case study from interior Alaska. Can. J. For. Res. **11**: 554–562.

List of symbols

$T_1, T_2, \ldots, T_M$	Time of the end of epochs 1, 2, ..., j
M	Number of epochs
$T = T_M$	Total length of period under study
N_j	Number of sample objects vulnerable in epoch j
n_j	Number of fires in epoch j
$n. = \sum_{j=1}^{M} n_j$	Total number of fires
$x_{j,r}$	Number of scars left by the rth fire in epoch j
$x.. = \sum_{j=1}^{M} \sum_{r=1}^{n_j} x_{j,r}$	Total number of scars
λ	Area-wide hazard of scarring
p	Probability that a fire leaves a scar on a given sample object
q	$1 - p$
$\theta = \lambda p$	Local hazard of scarring
$\mathrm{FI} = 1/\theta$	Fire interval: expected time between scars on a given sample object
ϕ	Overdispersion parameter
τ	Length of a generic epoch
$t_1, t_2, \ldots, t_n$	Times at which scars were left in generic epoch
Q	Quasi-likelihood
$\hat{\lambda}$, $\hat{q}$, etc.	MLE of λ, q, etc.
$V(q, N)$	Variance function (eq. 12)
$s_{\hat{\lambda}}$	Standard error of MLE $\hat{\lambda}$
$s_{\hat{q}}$	Standard error of MLE $\hat{q}$
$s_{\hat{\theta}}$	Standard error of MLE $\hat{\theta}$
$s_{\hat{\mathrm{FI}}}$	Standard error of MLE $\hat{\mathrm{FI}}$